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Planetary Migration in Protoplanetary Disks

A. Del Popolo

Dipartimento di Matematica, Università Statale di Bergamo, Piazza Rosate, 2 - I 24129 Bergamo, ITALY

Feza Gürsey Institute, P.O. Box 6 Çengelköy, Istanbul, Turkey

Abstract. In the current paper, we further develop the model for the migration of planets introduced in Del Popolo et al. (2001) and extended to time-dependent planetesimal accretion disks in Del Popolo & Ekşi (2002). We use a method developed by Stepinski & Valageas (1996, 1997), that is able to simultaneously follow the evolution of gas and solid particles for up to 10^7yr. The disk model is coupled to the migration model introduced in Del Popolo et al. (2001) in order to obtain the migration rate of the planet in the planetesimal disk. We find that in the case of disks having total mass of $10^{-3} - 0.1 M_\odot$, and $0.1 < \alpha < 0.0001$, planets can migrate inward a large distance while if $M < 10^{-3} M_\odot$ the planets remain almost in their initial position for $0.1 < \alpha < 0.01$ and only in the case $\alpha < 0.001$ the planets move to a minimum value of orbital radius of $\simeq$ 2AU. The model gives a good description of the observed distribution of planets in the period range 0-20 days.

1. Introduction

In order to study the formation of planetary systems it is necessary to study the global evolution of solid material which constitutes, together with gas, the protoplanetary disks. In the following, we introduce a time-dependent accretion disk model that shall be used in the next sections to study planets migration.

It is usually assumed that the surface density in planetesimals is proportional to that of gas (Lin & Papaloizou (1980, page 47); Murray et al. (1998)) and that the spatial structure and the time evolution of the surface density, $\Sigma_{\rm s}(R,t)$, is such that $\Sigma_{\rm s}(R,t) \propto \Sigma(R,t)$. Although this approximation has been and is widely used in literature, a better way to proceed is to directly calculate the evolution of solids in the disk. We know that the evolution of the surface density of gas is described by a diffusive-type equation, while that of solid particles is an advection-diffusion equation:

$$\frac{\partial \Sigma_{\rm s}}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}(\nu_{\rm s}\Sigma_{\rm s} r^{1/2})\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{2r\Sigma_s\langle\overline{v}_\phi\rangle_s}{\Omega_k t_s}\right] \tag{1}$$

where $\nu_{\rm s} = \frac{\nu}{\rm Sc}$, the Schmidt number, Sc, is given by:

$$\mathrm{Sc} = (1 + \Omega_{\rm k} t_{\rm s})\sqrt{1 + \frac{\overline{\mathbf{v}}^2}{V_{\rm t}^2}} \tag{2}$$

where $\mathbf{v}$ is the relative velocity between a particle and the gas, V_{t} the turbulent velocity, Ω_{k} is the Keplerian angular velocity, v_{ϕ} is the tangential velocity, and t_{s} the so called stopping time. Cylindrical polar coordinates (r, ϕ, z) are used.

Stepinski & Valageas (1996, 1997) developed a method that, using a series of simplifying assumptions, is able to simultaneously follow the evolution of gas and solid particles due to gas-solid-coupling, coagulation, sedimentation, and evaporation/condensation for up to 10^7yr.

The equation of gas evolution is solved by means of an implicit scheme and the evolution of gas is computed independently from the evolution of particles. At every time step the quantities needed for evaluating the change in the mass distribution of solids are calculated and the change itself is computed from Equation (1) using the operator splitting method. In such a method the advective term in Equation (1) is treated by the numerical method of characteristics, whereas an implicit scheme is applied to the diffusion term. The obtained distribution of solid material is then modifed because of the existence of the evaporation radius, and the mass distribution of the vapor is calculated using the implicit scheme. Finally, the new particle size distribution is calculated before proceeding to the next time step.

We suppose that a single planet moves in a planetesimal disk under the influence of the gravitational force of the sun. The equation of motion of the planet can be written as:

$$\ddot{\mathbf{r}} = \mathbf{F}_{\odot} + \mathbf{R} \tag{3}$$

(Melita & Woolfson 1996), where the term $\mathbf{F}_{\odot}$ represents the force per unit mass from the Sun, while $\mathbf{R}$ is the dissipative force (the dynamical friction term-see Melita & Woolfson 1996).

In order to take into account dynamical friction, we need a suitable formula for a disk-like structure such as the protoplanetary disk.

We assume that the matter-distribution is disk-shaped, then we have that:

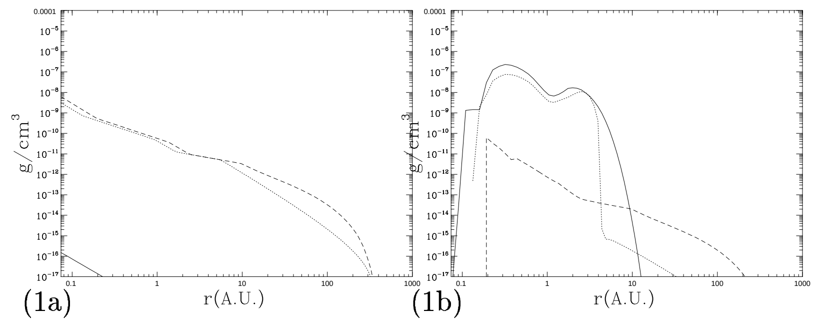
$$\mathbf{R} = -k_{||} v_{1||} \mathbf{e}_{||} - k_{\perp} v_{1\perp} \mathbf{e}_{\perp} \tag{4}$$

where $\mathbf{e}_{||}$ and $\mathbf{e}_{\perp}$ are two unit vectors parallel and perpendicular to the disk plane and $k_{||}$ and $k_{\perp}$ are given in Del Popolo & Ekşi (2002).

2. Results

Our model starts with a fully formed gaseous giant planet of $1 M_{\mathrm{J}}$ at 5.2 AU. According to several evidences showing that the disk lifetimes range from 10^5 yr to 10^7 yr (Strom et al. 1993; Ruden & Pollack 1991), we assume that the disk has a nominal effective lifetime of 10^6 years (Zuckerman et al. 1995). We integrated the model introduced in the previous section for several values of disk masses: $M_{\mathrm{D}} = 0.1$, 0.01, 0.001, 0.0001 $M_{\odot}$, and several values of α, where α is the dimensionless viscosity introduced by Shakura & Sunyaev (1973).

The results of the disk model are plotted in Figure 1. Figure 1a shows the evolution of the density of the gas component of the disk in the case $M = 0.1 M_{\odot}$ and $\alpha = 0.1$ (see figure). As shown in Figure 1b, in agreement with Stepinski & Valageas (1997) and Kornet et al. (2001), the most important result of the



(1a) (1b)

Figure 1. (a) Evolution of gas for a disc with $M_d = 0.1M_\odot$ and $\alpha = 0.1$ at $t = 10^4$yrs (dashed line), $t = 10^6$yrs (dotted line) and $t = 10^7$yrs (solid line). (b) Same as 1a but for solids.

low-mass models calculation (disks with $M < 0.1M_\odot$, and angular momentum $\simeq 10^{52}$g cm^2/s) is that such models lead to the survival of solid material, as can be seen from the evolution of the particle size, or by the emergence of the converged, nonvanishing surface density distribution of solids.

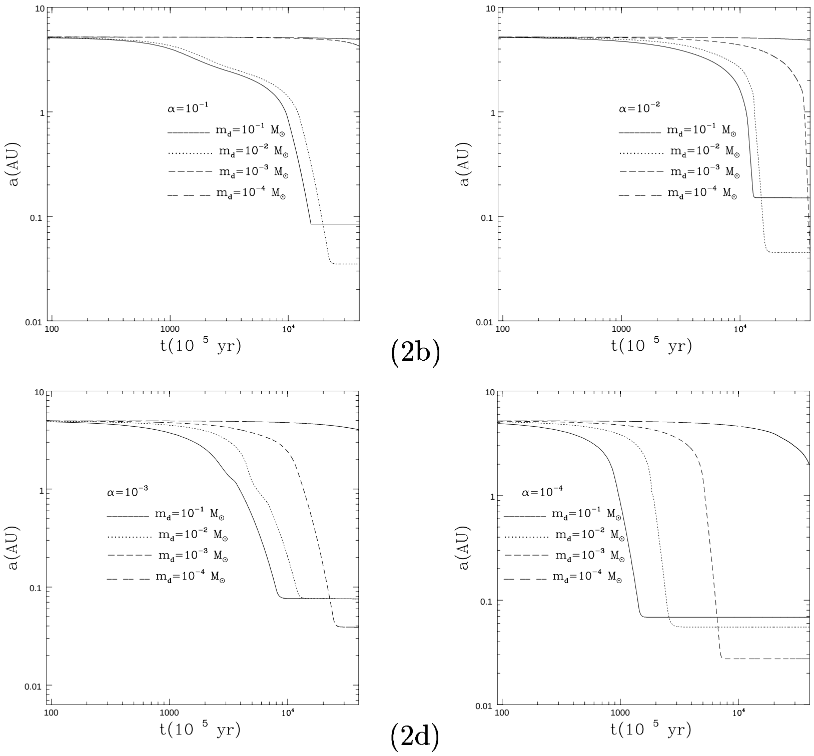
In the successive Figures (2a-2d), we plot the evolution of semi-major axis of the planet (see Figure). Summarizing Figures 2a-2d, according to the final distribution of planets distances, the present model predicts that, unless the disk mass is very small $M \simeq 0.0001M_\odot$, planets tend to move close to the central star to distances of the order of 0.03 AU. It is also evident that it is possible to find a planet at any distance from their locations of formation and very small distances from the parent star for peculiar values of the parameters α and M.

In Figure 3a, in order to show the predictions of the model for what concerns the distribution of planets in the inner part of the disc, we plot the fraction of planets in the orbital period range 0-20 days calculated using the model of this paper. Figure 3b, represents the same distribution obtained with the data given in www.exoplanets.org (see also Kuchner & Lecar 2002).

According to Kuchner & Lecar 2002, disk temperature determines the orbital radii of the innermost surviving planets, similarly to our model. As streseed in DP1, DP2, the model has not the drawback of Murray et al. 1998 model, namely that of requiring a too large disk mass for migration and at the same time has the advantage of Murray et al. 1998 model of having an intrinsic natural mechanism that provides halting of migration.

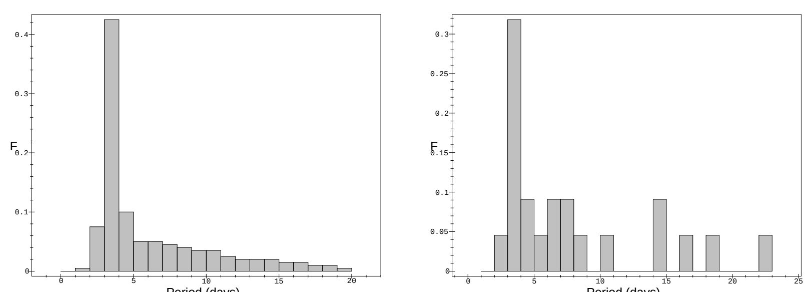
3. Conclusions

In the current paper, we further develop the model for the migration of planets introduced in Del Popolo et al. (2001) and extended to time-dependent planetesimal accretion disks in Del Popolo & Ekşi (2002). After releasing the assumption of Del Popolo & Ekşi (2002) that the surface density in planetesimals is proportional to that of gas, we used a method developed by Stepinski & Valageas (1996, 1997), using a series of simplifying assumptions, that is able to simultaneously follow the evolution of gas and solid particles due to gas-solid-coupling, coagulation, sedimentation, and evaporation/condensation for up to



(2a) (2b) (2c) (2d)

Figure 2. (a) The evolution of the semi-major axis $a(t)$ of a jupiter-mass planet, $M = 1M_{\mathrm{J}}$, in a planetesimal disk for $\alpha = 0.1$ and several values of M_{d}, 0.1 (solid line), 0.01 (dotted line), 0.001 (short-dashed line) and $0.0001M_{\odot}$ (long-dashed line). (b) Same as Fig. 2a but with $\alpha = 0.01$. (c) Same as Fig. 2a but with $\alpha = 0.001$. (d) Same as Fig. 2a but with $\alpha = 0.0001$.



(3a) (3b)

Figure 3. (a) Fraction of planets having orbital periods in the range 0-20 days, calculated according the model of this paper. (b) Fraction of planets having orbital periods in the range 0-20 days, calculated according to data (see www.exoplanets.org).

10^7yr. Then we coupled the disk model to the migration model introduced in Del Popolo et al. (2001) in order to obtain the migration rate of the planets in the planetesimal disk and to study how the migration rate depends on the disk mass, on its time evolution and on some values of the dimensionless viscosity parameter α. We found that in the case of disks having total mass of $10^{-3} - 0.1 M_{\odot}$, and $0.1 < \alpha < 0.0001$, planets can migrate inward a large distance while if $M < 10^{-3} M_{\odot}$ the planets remains almost in their initial position for $0.1 < \alpha < 0.01$ and only in the case $\alpha < 0.001$ the planets move to a minimum value of orbital radius of $\simeq 2$ AU. The model was also used to calculate the distribution of planets in the inner part of the disc (0-0.14 AU). The result showed that the planets distribution obtained by means of the model of the present paper gives a good description of the observed distribution of planets in the period range 0-20 days.

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